Lab 5: Simple Linear Regression

Week 14

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Plan

In this lab we will practice:

1. Visualizing linear relationships
2. Estimating and interpreting **simple linear regression models**
3. Computing slope and intercept **manually and using R**
4. Evaluating **model fit (R^2)** and **residuals**
5. Reflecting on **prediction vs. causality**

Textbook Reference: JA Chapter 17

Warm-up & Review

Think about:

- What does the slope represent in a regression line?
- Does correlation imply causation?
- Why do we square residuals in OLS?

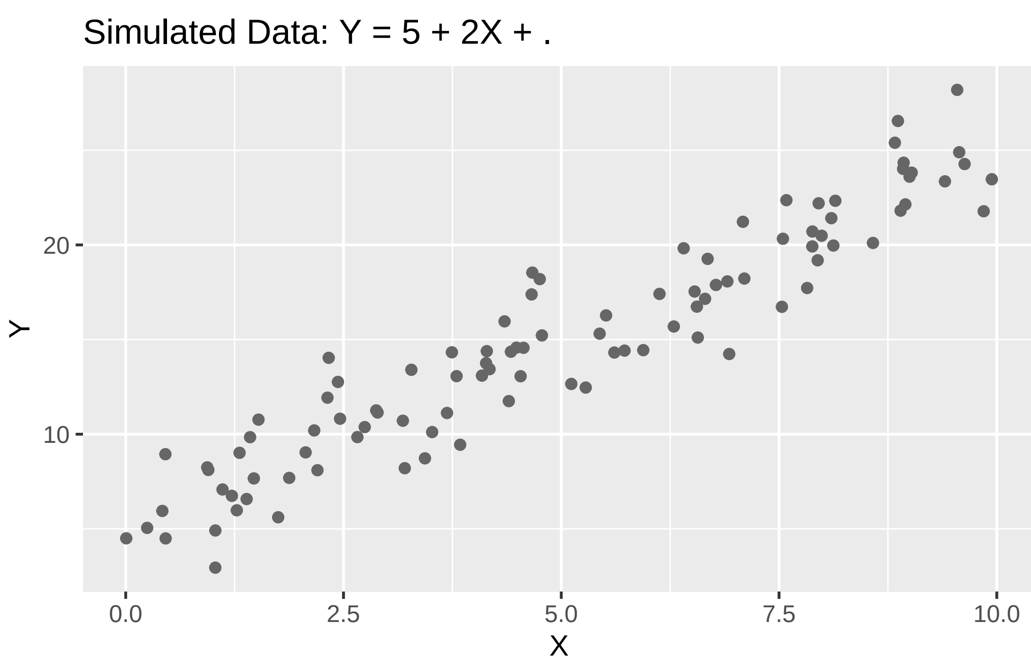
Exercise 1: Visualizing a Linear Relationship

Simulated data

```
set.seed(123)
n <- 100
x <- runif(n, 0, 10)
y <- 5 + 2*x + rnorm(n, 0, 2)
simdata <- tibble(x, y)
```

Exercise 1: Visualizing a Linear Relationship

```
ggplot(simdata, aes(x=x, y=y)) +
  geom_point(color="grey40") +
  labs(title="Simulated Data: Y = 5 + 2X +  ",
       x="X", y="Y")
```



Task 1

> Caution
>
> 1. What sign do you expect for the correlation between `x` and `y`?
> 2. Add a fitted line using `geom_smooth(method="lm")` and confirm visually.

Exercise 2: Manual OLS Estimation

Compute slope and intercept manually using formulas:

$$\hat{\beta} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}, \qquad \hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}.$$

```
beta_hat <- cov(simdata$x, simdata$y) / var(simdata$x)
alpha_hat <- mean(simdata$y) - beta_hat * mean(simdata$x)
c(alpha_hat, beta_hat)
```

```
[1] 4.982080 1.982034
```

Exercise 2: Manual OLS Estimation

Compare with R's built-in estimator:

```
model_sim <- lm(y ~ x, data = simdata)
summary(model_sim)
```

```
Call:
lm(formula = y ~ x, data = simdata)

Residuals:
    Min      1Q  Median      3Q     Max
```

```
-4.4759 -1.2265 -0.0395  1.1927  4.4345

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4.98208    0.39211   12.71   <2e-16 ***
x            1.98203    0.06836   28.99   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.939 on 98 degrees of freedom
Multiple R-squared:  0.8956,	Adjusted R-squared:  0.8945 
F-statistic: 840.6 on 1 and 98 DF,  p-value: < 2.2e-16
```

Task 2

- Interpret the slope: what does a one-unit increase in X imply for Y?
- How close are your manual and R estimates? Why are they identical (up to rounding)?

Exercise 3: Regression with CPS Data

Question: How does education relate to weekly earnings?

```r
data(cps)
model_cps <- lm(earnwk ~ educ, data = cps)
summary(model_cps)
```

```

Call:
lm(formula = earnwk ~ educ, data = cps)

Residuals:
    Min      1Q  Median      3Q     Max 
-1272.1  -417.3  -157.4   229.1  7282.6 

Coefficients:
```

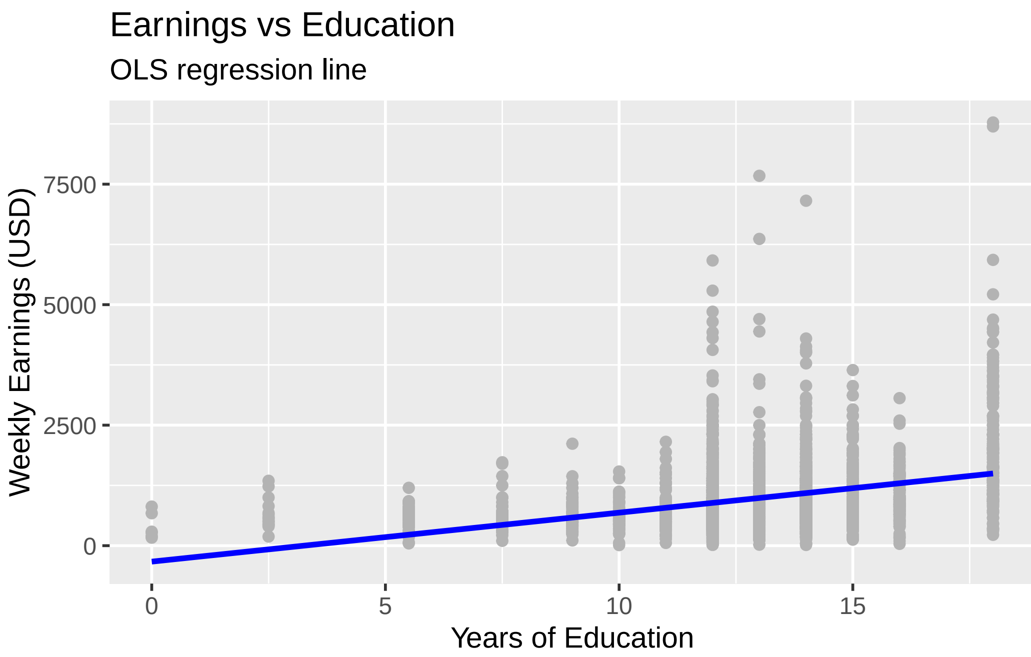
```
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -330.753     72.713  -4.549 5.63e-06 ***
educ         101.550      5.575  18.217  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 709.8 on 2807 degrees of freedom
  (1204 observations deleted due to missingness)
Multiple R-squared:  0.1057,    Adjusted R-squared:  0.1054
F-statistic: 331.9 on 1 and 2807 DF,  p-value: < 2.2e-16
```

Task 3

1. Interpret the slope: how much does weekly earnings increase per year of education?
2. Is the intercept meaningful here?
3. Report R^2 and explain what it measures.

Exercise 4: Visualizing the Fit

```r
ggplot(cps, aes(x=educ, y=earnwk)) +
  geom_point(color="grey70") +
  geom_smooth(method="lm", se=FALSE, color="blue") +
  labs(title="Earnings vs Education",
       subtitle="OLS regression line",
       x="Years of Education", y="Weekly Earnings (USD)")
```



Task 4

- Add residual lines with `geom_segment()`.
- Identify one observation with a large positive and one with a large negative residual.
- What could explain them?

Exercise 5: Prediction and Causality

Use the fitted model to predict average earnings for 12, 14, and 16 years of education.

```
predict(model_cps, newdata = data.frame(educ = c(12, 14, 16)))
```

```
        1         2         3 
 887.8488 1090.9491 1294.0494 
```

Task 5

- What happens to predicted earnings when education increases by 2 years?
- Can we interpret this as a **causal effect** of education on income? Why or why not?
- What omitted factors might bias the estimate?

Challenge Problem

Simulate a new dataset where $Y = 5 + 2X + U$ but U is correlated with X (e.g., `U <- 0.5*X + rnorm(n)`).

Estimate the regression again and compare the slope.

Question: Does the estimated slope still recover the true value 2? Why not?

Exit Question

> Under what condition can we interpret the slope $\hat{\beta}$ as a **causal effect**?

Submission

Submit the rendered PDF or HTML report on Canvas as a group.
Be sure to include your plots, coefficient outputs, and short written interpretations.