HW 01: Probability and Conditional Probability

Practice, practice, and practice

> **❗ Due date**
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> This assignment is due on **Monday, September 1 at 11:59pm**. To be considered on time, the following must be done by the due date:
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> - Final `.pdf` file submitted on Gradescope

Introduction

This first homework is fully conceptual, where you will practice your skills in applying the concept of probability and conditional probability. The next homework might contain an applied exercise where you will practice using R code and Quarto.

Learning goals

In this assignment, you will...

- apply the concept of event and sample space to compute probability
- apply various concepts about event operation, such as union, intersection, disjoint, and complement.
- use the axioms of probability and the additive rule to compute the probability of an event
- apply the concept of conditional probability

Conceptual exercises

Instructions

The conceptual exercises focus on explaining concepts and showing results mathematically. Show your work for each question. These problems are from SDG's exercises on page 21. While a solution manual may exist out there, I expect you not to skip any steps and carefully understand the thinking process when working on these exercises.

You may write the answers and associated work for conceptual exercises by hand or type them in your Quarto document.

Exercise 1 (6 points)

One ball is to be selected from a box containing red, white, blue, yellow, and green balls. If the probability that the selected ball will be red is $1/5$ and the probability that it will be white is $2/5$.

1. Write down the sample spaces for this experiment (there are several ways to do this) **(+1 points if you can come up with more than one way)**
2. What is the probability that it will be red or white?
3. What is the probability that it will be blue, yellow, or green?

Exercise 2 (2 points)

A student selected from a class will be either a boy or a girl. If the probability that a boy will be selected is 0.3. What is the probability that a girl will be selected?

Exercise 3 (7 points)

Consider two events A and B such that $Pr(A) = 1/3$ and $Pr(B) = 1/2$. Determine the value of $Pr(A \cap B^c)$ for the following conditions:

1. If A and B are disjoint
2. If $A \subset B$
3. If $Pr(A \cap B) = 1/8$

Exercise 4 (7 points)

If the probability that student A will fail a certain statistics examination is 0.5, the probability that student B will fail the examination is 0.2, and the probability that both student A and student B will fail the examination is 0.1.

1. What is the probability that at least one of these two students will fail the examination?
2. Can you compute the probability that both students will pass the exam? If yes, please find the probability; if not, why?
3. What is the probability that exactly one student will fail the exam?

Exercise 5 (6 points)

Consider two events A and B with $Pr(A) = 0.4$ and $Pr(B) = 0.7$. Determine the maximum and minimum possible values of $Pr(A \cap B)$ and the conditions under which each of these values is attained.

Exercise 6 (7 points)

Solve the following problem to practice the concept of conditional probability.

1. If $A \subset B$ with $Pr(B) > 0$, what is the value of $Pr(A|B)$?
2. If A and B are disjoint events and $Pr(B) > 0$, what is the value of $Pr(A|B)$?
3. If S is the sample space of an experiment and A is any event in that space, what is the value of $Pr(A|S)$?

Exercise 7 (7 points)

Each time a shopper purchases a tube of toothpaste, he chooses either brand A or brand B. Suppose that for each purchase after the first, the probability is $1/3$ that he will choose the same brand that he chose on his preceding purchase, and the probability is $2/3$ that he will switch brands.

1. Let A_i denote the event that the shopper purchases brand A on his i-th purchase, similarly for B_i, for $i = 1, 2, 3, ...$ List the known probability stated above using this notation.
2. If he is equally likely to choose either brand A or brand B on his first purchase, what is the probability that both his first and second purchases will be brand A and both his third and fourth purchases will be brand B?

Exercise 8 (7 points)

A box contains three cards. One card is red on both sides, one card is green on both sides, and one card is red on one side and green on the other. One card is selected from the box at random, and the color on one side is observed. If this side is green, what is the probability that the other side of the card is also green?

Submission

> ⚠️ Warning
>
> Remember – you must turn in a PDF file to the Gradescope page before the submission deadline for full credit.
> Instructions to combine PDFs:
>
> - Preview (Mac): support.apple.com/guide/preview/combine-pdfs-prvw43696/mac
>
> - Adobe (Mac or PC): helpx.adobe.com/acrobat/using/merging-files-single-pdf.html

To submit your assignment:

- Access Gradescope through the menu on the ECON2250 Canvas site.

- Click on the assignment, and you'll be prompted to submit it.

- Mark the pages associated with each exercise. All of the pages of your HW **must** be associated with at least one question (i.e., should be "checked").

- Select the first page of your PDF submission to be associated with the *"Formatting"* section.

Grading (50 points)

Component	Points
Ex 1	6
Ex 2	2
Ex 3	7
Ex 4	7
Ex 5	6
Ex 6	7
Ex 7	7

Component	Points
Ex 8	7
Formatting	1

Formatting" grade is to assess whether or not you mark specific page of the pdf to a specific questions to help the TA when grading.